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COCKPIT AUTOMATIC PRESSURE REGULATION USING DIRECT ACTION AIRFLOW REGULATION SYSTEM

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Introduction

Automatic regulation of cockpit airflow (Q_t) has the following purposes:

1) Counterbalance of ejected air (through Automatic Regulation System-ARS-exhaust valve and through non-sealed parts of the cabin).

2) Cabin air pressure regulation.

The ARS consists from: airflow sensor, regulator and working element.

System structure:

Such an ARS is depicted in Fig. 1 [1, 2] where:

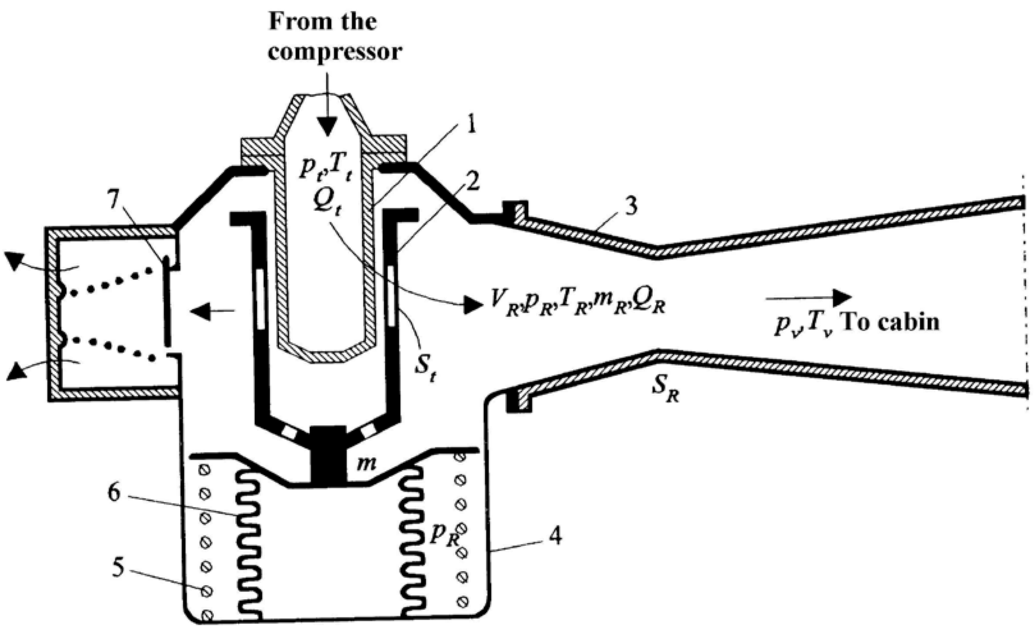

Fig. 1. The structure of the ARS for the cabin airflow.
1 – fixed cylinder; 2 – mobile cylinder with valve; 3 – Venturi tube (exhaust); 4 – corps; 5 – spring; 6 – gophred box; 7 – safety valve.

On the side of the fixed cylinder there are several controlled exhausts. The status of those exhausts is linked (and controlled) by the position of the mobile cylinder, thus by valve *2*. The motion of valve *2* is controlled by elements *5* and *6*. If pressure p_a increases over a prior established maximum value, the air is evacuated into the atmosphere through the safety valve (7).

The regulator equation:

The ARS (as presented in Fig. 1) is holding p_R = constant (the air pressure within the regulator chamber) for limiting the amount of air transmitted to the cabin. After applying the derivative with time, (where V_R = volume of regulator's chamber) we have:

$$V_R \cdot p_R = m_R R T_R \tag{1}$$

and, assuming $\frac{dT_R}{dt} = 0$ (variation rate of the temperature within the regulator chamber is negligible) we have:

$$V_R \frac{dp_\mathrm{R}}{dt} = RT_R \frac{dm_R}{dt}, \frac{dm_R}{dt} = Q_t - Q_R. \tag{2}$$

The flows Q_t and Q_R are functions depending on parameters mentioned in Fig. 1, thus [3]:

$$Q_t = f_t\left(p_t, p_R, S_t, T_t\right), \tag{3}$$

$$Q_R = f_R\left(p_R, p_\mathrm{v}, S_R, T_R\right). \tag{4}$$

Using Taylor series, negliging non-linear terms (superior order small infinities), and assuming $S_R = \text{constant}$, we will have :

$$\Delta Q_t = \left(\frac{\partial Q_t}{\partial p_t}\right)_0 \Delta p_t + \left(\frac{\partial Q_t}{\partial p_R}\right)_0 \Delta p_R + \left(\frac{\partial Q_t}{\partial S_t}\right)_0 \Delta S_t + \left(\frac{\partial Q_t}{\partial T_t}\right)_0 \Delta T_t, \tag{5}$$

$$\Delta Q_R = \left(\frac{\partial Q_R}{\partial p_R}\right)_0 \Delta p_R + \left(\frac{\partial Q_R}{\partial p_\mathrm{v}}\right)_0 \Delta p_\mathrm{v} + \left(\frac{\partial Q_R}{\partial T_R}\right)_0 \Delta T_R. \tag{6}$$

Substituting (5), (6) in (2) we have the regulator dynamic equation:

$$\begin{gathered}\frac{V_R}{RT_R^0}\frac{\mathrm{d}p_R}{\mathrm{d}t} + \left(\frac{\partial Q_R}{\partial p_R} - \frac{\partial Q_t}{\partial p_R}\right)_0 \Delta p_R = \left(\frac{\partial Q_t}{\partial p_t}\right)_0 \Delta p_t - \left(\frac{\partial Q_R}{\partial p_\mathrm{v}}\right)_0 \Delta p_\mathrm{v} + \\ + \left(\frac{\partial Q_t}{\partial T_t}\right)_0 \Delta T_t - \left(\frac{\partial Q_R}{\partial T_R}\right)_0 \Delta T_R + \left(\frac{\partial Q_t}{\partial S_t}\right)_0 \Delta S_t\end{gathered} \tag{7}$$

than

$$\begin{gathered}\frac{\Delta p_R}{p_N} = \overline{p}_R; \frac{\Delta p_t}{p_N} = \overline{p}_t; \frac{\Delta p_\mathrm{v}}{p_N} = \overline{p}_\mathrm{v}; \frac{\Delta Q_R}{Q_{t\max}} = q_R, \\ \frac{\Delta Q_t}{Q_{t\max}} = q_t; \frac{\Delta T_t}{T_{t\max}} = \theta_t; \frac{\Delta T_R}{T_{R\max}} = \theta_R; \frac{\Delta S_t}{S_{t\max}} = \overline{S}_t\end{gathered} \tag{8}$$

We will abtain non-dimensional equation (9) where (10) and zero indexed parentheses were omitted.

$$\tau_R^* \frac{d\overline{p}_R}{dt} + k_R \overline{p}_R = \frac{\partial q_t}{\partial \overline{p}_t}\overline{p}_t - \frac{\partial q}{\partial \overline{p}_v}\overline{p}_v + \frac{\partial q_t}{\partial \theta_t}\theta_t - \frac{\partial q_R}{\partial \theta_R}\theta_R + \frac{\partial q_t}{\partial \overline{S}_t} \cdot \overline{S}_t, \tag{9}$$

$$\tau_R^* = \frac{V_R p_N}{RT_R^0 Q_{t\max}}, \tag{10}$$

$$k_R = \frac{\partial q_R}{\partial \overline{p}_R} - \frac{\partial q_t}{\partial \overline{p}_R}, \tag{11}$$

where τ_R^* is the regulator chamber filling time and $k_R =$ working element pressure autoequalizing coefficient.

Mobile elements equation

In stationary regime, the flow regulator valve is an equilibrium position, namely x_0 coordinate (where *x* cylinder *2* moment positive for downwards moves) [4].

The forces considerated an mobile elements (having m mass) ave.:

$$S_{ef}\, p_R = F_i + F_\nu + F_e + F_f, \tag{12}$$

where S_{ef} = effective area of gophred box; F_i = inertia force; $F_\nu = \eta \frac{\mathrm{d}x}{\mathrm{d}t}$ = viscosity friction force; $F_e = k_e x$ = elasticity force (for the grind $-k_e$ = elasticity coefficient); F_f = dry friction force.

Using the above relations, (12) become:

$$S_{ef}\Delta p_R = m\frac{d^2\Delta x}{dt^2} + \eta\frac{d\Delta x}{dt} + k_e\Delta x + F_f. \tag{13}$$

Assuming $F_f = 0$ and introducing dimensionless value.

$$\overline{x} = \frac{\Delta x}{x_{\max}} = -\frac{\Delta S_t}{S_{t\max}} = -\overline{S}_t \tag{14}$$

we have:

$$\tau_m^2 \frac{d^2\overline{S}_t}{dt^2} + 2\xi_m\tau_m\frac{d\overline{S}_t}{dt} + \overline{S}_t = -k\overline{p}_R, \tag{15}$$

where:

$$\tau_m^2 = \frac{m}{k_e}, 2\xi_m\tau_m = \frac{\eta}{k_e}, k = \frac{S_{ef}\cdot p_N}{k_e x_{\max}}. \tag{16}$$

The System Mathematical Model. Stability analysis.

Having in (9)

$$\frac{\partial q_t}{\partial \overline{p}_t}\overline{p}_t - \frac{\partial q_R}{\partial \overline{p}_\nu}\overline{p}_\nu + \frac{\partial q_t}{\partial \theta_t}\theta_t - \frac{\partial q_R}{\partial \theta_R}\theta_R = F(t), \tag{17}$$

$$\frac{\partial q_t}{\partial \overline{S}_t} = k_t \tag{18}$$

(9) end (15) become:

$$\tau_R\frac{d\overline{p}_R}{dt} + \overline{p}_R - \frac{k_t}{k_R}\cdot\overline{S}_t = \frac{1}{k_R}\cdot F(t), \tag{19}$$

$$\tau_m^2 \frac{d^2\overline{S}_t}{dt^2} + 2\xi_m\tau_m\frac{d\overline{S}_t}{dt} + \overline{S}_t = -k\overline{p}_R \tag{20}$$

with $\tau_R = \frac{\tau_R^*}{k_R}$ = time constant.

After the application of Laplace operator with zero initial conditions and substituting $\overline{S}_t(s)$ we have:

$$\left(a_3s^3 + a_2s^2 + a_1s + a_0\right)\overline{p}_R(s) = \left(\tau_m^2s^2 + 2\xi_m\tau_m s + 1\right)F(s), \tag{21}$$

where

$$a_3 = k_R \tau_R \tau_m^2, a_2 = k_R\left(2\xi_m \tau_m \tau_R + \tau_m^2\right),$$
$$a_1 = k_R\left(\tau_R + 2\xi_m \tau_m\right), a_0 = k_R + kk_t. \tag{22}$$

The system characteristic equation is:

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \tag{23}$$

and Hurwitz stability conditions are exprimed by making the coefficients a_i , $i = \overline{0,3}$ positive and

$$a_1 a_2 > a_0 a_3 \tag{24}$$

which, law (22) is:

$$k_R\left(\tau_R + 2\xi_m \tau_m\right)\left(\tau_m + 2\xi_m \tau_R\right) > \tau_R \tau_m\left(k_R + kk_t\right). \tag{25}$$

In Fig. 2 we have the block diagram (with transfer functions) of the model described by (19) and (20).

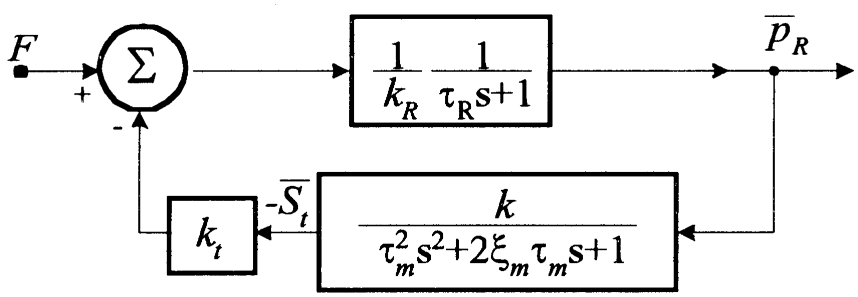

Fig. 2. Block diagram with transfer functions of the components of the ARS.

Bassed on equation (21) in stationary regime

$$\overline{p}_R(\infty) = \frac{1}{k_R + kk_t} F(\infty). \tag{26}$$

For *Q_{tmax} = 550 kg/h = 0,153 kg/s, R = 287 J/kgK, t_R = 27 ^{0}C (T_R = 300 K), V_R = 100 cm^3 = 10^{-4} m^3* and *p_N = 10^5 N/m,* with relation (10), we have:

$$\tau_R^* = 7,6 \cdot 10^{-3} \mathrm{s}.$$

Going back to dimensioned relations (11) we have:

$$k_R = \left(\frac{\partial q_R}{\partial \overline{p}_R} - \frac{\partial q_t}{\partial \overline{p}_R}\right)_0 = \frac{p_N}{Q_{t\max}}\left(\frac{\partial(\Delta Q_R)}{\partial(\Delta p_R)} - \frac{\partial(\Delta Q_t)}{\partial(\Delta p_R)}\right)_0 = \frac{p_N}{Q_{t\max}}\left(\frac{\partial Q_R}{\partial p_R} - \frac{\partial Q_t}{\partial p_R}\right) \approx$$
$$\approx \frac{p_N}{Q_{t\max}}\left(\frac{\Delta Q_R - \Delta Q_t}{\Delta p_R}\right)_0 = \frac{p_N}{\Delta p_R}\left(\frac{\Delta Q_R}{Q_{t\max}} - \frac{\Delta Q_t}{Q_{t\max}}\right)_0$$

For $\Delta Q_R / Q_{t\max} = 4 \cdot 10^{-3}, \Delta Q_t / Q_{t\max} = -6 \cdot 10^{-3}, \Delta p_R / p_N = 10^{-2}$ we have $k_R = 1$, and $\tau_R = \tau_R^* / k_R = 7,6 \cdot 10^{-3} \mathrm{s}$.

Coefficient k_t can be computed with relation (18)

$$k_t = \left(\frac{\partial q_t}{\partial \bar{S}_t}\right)_0 \approx \left(\frac{\Delta Q_t}{Q_{t\max}}\right)_0 \left(\frac{S_{t\max}}{\Delta S_t}\right)_0 = -\left(\frac{\Delta Q_t}{Q_{t\max}}\right)_0 \left(\frac{x_{\max}}{\Delta x}\right)_0,$$

which, for $\Delta x / x_{\max} = 10^{-1}$ and $\Delta Q_t / Q_{t\max} = -6 \cdot 10^{-3}$ has the following value $k_t = 6 \cdot 10^{-2}$.

For $m = 100\text{g} = 10^{-1}\text{kg}, k_e = 1\text{N/m}, \eta = 3,2 \cdot 10^{-1}\text{Ns/m}, x_{\max} = 1,5\text{cm} = 1,5 \cdot 10^{-2}\text{m}$ and $S_{ef} = 3 \cdot 10^{-3}\text{m}^2$, with (16) we have $\tau_m \approx 0,32, \xi_m = 0,5$ and $k = 2 \cdot 10^4$.

Substituting those parameters in (22) we determine coefficients a_3, a_2, a_1, a_0 whose values verify the Hurwitz stability conditions.

Going back to dimensioned relation (17) we have [5]:

$$F(t) = \left(\frac{\partial q_t}{\partial \bar{p}_t}\bar{p}_t + \frac{\partial q_t}{\partial \theta_t}\theta_t - \frac{\partial q_R}{\partial \bar{p}_\nu}\bar{p}_\nu - \frac{\partial q_R}{\partial \theta_R}\theta_R\right)_0 = \frac{1}{Q_{t\max}}\left(\frac{\partial Q_t}{\partial p_t}\Delta p_t + \frac{\partial Q_t}{\partial T_t}\Delta T_t\right)_0 -$$

$$-\frac{1}{Q_{t\max}}\left(\frac{\partial Q_R}{\partial p_\nu}\Delta p_\nu + \frac{\partial Q_R}{\partial T_R}\Delta T_R\right)_0 = \left(\frac{\Delta Q_t}{Q_{t\max}}\right)_{\substack{P_R=\text{const.}\\ S_t=\text{const.}}} - \left(\frac{\Delta Q_R}{Q_{t\max}}\right)_{P_R=\text{const.}}$$

Upon (26), in stationary regime:

$$\bar{p}_R(\infty) = \frac{1}{k_R + kk_1}F(\infty) = \frac{2 \cdot 10^{-3}}{1 + 2 \cdot 10^4 \cdot 6 \cdot 10^{-2}} \approx 0,17 \cdot 10^{-5}$$

and

$$\Delta p_R(\infty) = \bar{p}_R(\infty)\, p_N = 0,17 \cdot 10^{-5} \cdot 10^5 = 0,17\text{N/m}^2 << 10^5\text{N/m}^2.$$

Conclusions:

In the paper we presented a study upon an ARS for the cabin airflow, using a direct action regulation system. The mathematical model for the system was written in there forms: dimensioned, dimensionless and operational.

The stability of the system is analyzed through the calculus of its parameters.

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Summary

For an ARS of the cabin airflow with direct action we built a linearised mathematical model in there forms: dimensioned, dimensionless and operational. Using the mathematical model, the system stability is analyzed through the calculus of the dynamic parameters of the automatic regulation system (ARS).